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THE HARDEST MATH PROBLEM
GRADE 8

The leaders of the EARTH Club learned a lot from the research they collected about bees. Now they are making plans to turn the facts into action.

"I bet a lot of students don't know that honeybees pollinate about one-third of the nation's crops, but are disappearing at an alarming rate," says Maria.

"Yeah, and pesticides are a big part of the problem," adds Vishal. "The bees need our help!"

The EARTH Club decides to launch a few exciting projects at their school to create buzz about saving the bees. It's a lot of work for the EARTH Club, and they're counting on math help from their newest member—you!

Solve the Problem

The 8th graders are volunteering to make and sell organic pesticides, called Sweet4Bees, that will protect plants in local gardens without harming bees. The teens also plan to donate some of their product to local nursing homes to use in their gardens. Jade is so excited! She's already started a business log in her notebook, including a graph with a break-even point of 34 bottles sold.

Suppose all Jade's notes become true. **What is the club's overall profit per ounce of pesticide solution, when 26 bottles have been donated to the nursing homes? Round to the nearest cent.**

Sweet4Bees Business Log

Production Expenses: $1.45 (empty bottle), $0.05 (label), cost of ingredients (not sure right now, I'll call it $*m*)

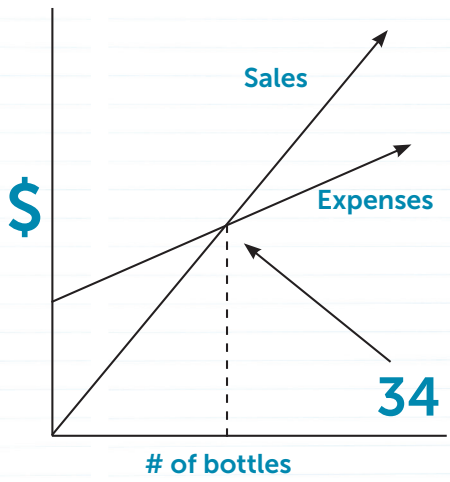
Taxes & Fees: none!

Selling Cost: $9.72/bottle

Start-Up Expenses: $65.88 (high-speed mixer), $55 (pans), $26 (kitchenware)

- Each bottle holds 12 oz of pesticide concentrate.* Directions for preparing solution: Mix 7 parts water to 1 part Sweet4Bees concentrate to create the pesticide solution.
- When profits > $500, for every 15 bottles sold, donate 2 bottles to local nursing homes for their gardens.

Sweet4Bees!



*concentrate = a stronger version of a liquid product, intended to be diluted with water to make its final form

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CHALLENGE 2 ANSWER KEY — GRADE 8

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I need to find the **"total overall profit,"** so I start with the profit equation.

Profit = Sales – Expenses

Step 2: Next, I set up the equations for **sales** and **expenses**.

Let x = number of bottles Let m = cost of making enough concentrate to fill 1 bottle

Each bottle is sold for $9.72, so: **Total Sales = $9.72x**

Total Expenses consist of variable expenses and fixed costs.

Variable expenses are those that *vary* according to the number of items made.

Fixed costs are the start-up, or one-time costs.

variable expenses *fixed costs*

Total Expenses = (1.45x$ + 0.05x$ + (m)x) + ($65.88 + $55 + $26)

(1.45x$): empty bottle; 0.05x$: label; (m)x: cost ($) of pesticide concentrate; $65.88: mixer; $55: pans; $26: kitchenware

Total Expenses = 1.50x$ + mx$ + $146.88

Step 3: Now, I'll look for the break-even point in my data or graph. This is the point where there is no gain or loss. In other words, sales equal expenses.

On a graph, it's the point where the two rays or lines intersect.

Jade's sketch shows the rays intersecting at 34, so this is the break-even point.

Step 4: I substitute the value of x into my equation to solve for m, the cost of making enough concentrate to fill 1 bottle.

I substitute the break-even point, x = 34.

$$\text{Sales} = \text{Expenses}$$
$$\$9.72(34) = \$1.50(34) + m(34) + 146.88$$
$$\$330.48 = \$51 + 34m + 146.88$$
$$\$330.48 = 34m + \$197.88$$
$$\$132.60 = 34m$$
$$\$3.90 = m$$

The cost of making just the concentrate is $3.90/bottle. I have to add in the empty bottle and label, too.

$3.90 + $1.45 + $0.05 = $5.40 cost to make 1 complete bottle

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Step 5: When profits are greater than $500, they will donate 2 bottles for every 15 sold. So, I need to find out how many bottles it takes to get to a profit greater than $500.

$$\textit{Profit} > \$500$$
$$\textit{Sales} - \textit{Expenses} > \$500$$
$$\$9.72x - [(\$5.40x + \$146.88)] > \$500$$
$$\$9.72x - \$5.40x - \$146.88 > \$500$$
$$\$4.32x - \$146.88 > \$500$$
$$\$4.32x > \$646.88$$
$$x > 149.\overline{740}$$

Starting with the 150th bottle, they need to donate 2 bottles for every 15 bottles sold.

Step 6: I use another equation to find when they reach a total donation of 26 bottles.

I translate the words into an equation: They donate **26** bottles, which **equals 2 times** the **number of times 15 occurs between** the milestone of **150** bottles sold and the future milestone of an unknown number of bottles sold, ***y***. While I'm setting up the equation, I also realize that subtracting 150 from *y* only provides the difference between those numbers (the number of bottles sold AFTER the 150th bottle). I need to account for the first bottle sold in the club's donation plan (the 150th bottle), so I add the +1 below.

$$\textit{donations} = 2\left(\frac{(y-150)+1}{15}\right)$$

$$26 = 2\left(\frac{y-149}{15}\right)$$
$$13 = \frac{y-149}{15}$$
$$195 = y - 149$$
$$344 = y$$

When they've sold the 344th bottle produced, they will have donated 26 bottles to the nursing homes.

$$\text{Profit} = \text{Sales} - \text{Expenses} - \text{Sales of 26 bottles}$$
$$\text{Profit} = \$9.72(344) - [\$5.40(344) + 146.88)] - \$9.72(26)$$
$$\text{Profit} = \$3{,}343.68 - [\$1{,}857.60 + 146.88] - \$252.72$$
$$\text{Profit} = \$3{,}343.68 - [\$2{,}004.48] - \$252.72$$
$$\text{Profit} = \$1{,}339.20 - \$252.72$$
$$\text{Profit} = \$1{,}086.48$$

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Step 7: Now, I'm ready to find the profit after selling the 344th bottle. Special note: No money was received for 26 of those bottles. I have to subtract that money in my calculations.

Step 8: I must figure out the profit per ounce of solution (not concentrate).
For each 12 oz bottle of concentrate, you need to mix 7 parts water to 1 part concentrate to get the actual amount of solution it will make. I set up and solve a proportion.

$$\frac{12 \text{ ounces con.}}{n \text{ ounces in total}} = \frac{1}{7+1} \qquad \frac{12 \text{ ounces con.}}{n \text{ ounces in total}} = \frac{1}{8} \qquad n = 12(8) \qquad n = 96$$

Each 1 bottle of concentrate makes 96 ounces of pesticide solution.

$$\frac{\text{profit}}{\text{ounce}} = \frac{\$1{,}086.48}{344 \text{ bottles}} * \frac{1 \text{ bottle}}{96 \text{ ounces of solution}} = \frac{\$0.032899709\ldots}{1 \text{ oz solution}}$$

Answer: Their overall profit, per ounce of pesticide solution, when 26 bottles have been donated to the nursing homes, is **about $0.03 (or 3 cents) per ounce**.

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