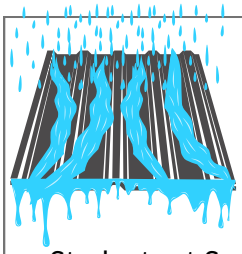


Try Your

THE HARDEST MATH PROBLEM

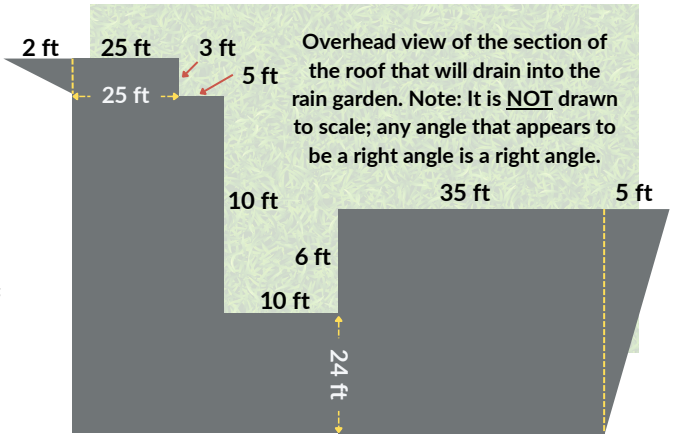
GRADE 7

Students at South Haven Middle School (SHMS) have been learning about ways to improve community resilience, particularly addressing risks from stormwater runoff. Runoff is water from rain or melting snow that flows over the land surface instead of soaking into the ground. It can carry pollutants like oil, trash, fertilizers, and sediment into nearby rivers, lakes, and storm drains, contributing to water pollution and erosion.

"Rain gardens are shallow, planted landscape depressions that temporarily hold and filter stormwater runoff, allowing it to soak into the ground," guest speaker Dr. Ailem Carvajal shared at the SHMS assembly. "They are filled with deep-rooted native plants and grasses which tolerate both wet and dry conditions. The size of a rain garden will depend on the size of the impervious area that will drain into it and the infiltration rate of the soil. An impervious area is any surface (such as a roof, paved sidewalk, or road) that does not allow water to soak into the ground."

Solve the Problem

Three students at SHMS, Destiny, Kass, and Huan, have permission from the principal to create a plan for a rain garden in a small section of the school yard. They want to control some of the flooding they have noticed from stormwater runoff flowing from a particular section of the school roof. The students acquired the dimensions for the section of the roof that would drain into their rain garden.



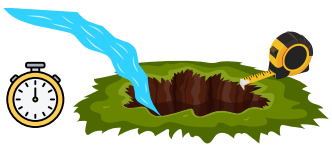
Overhead view of the section of the roof that will drain into the rain garden. Note: It is NOT drawn to scale; any angle that appears to be a right angle is a right angle.

"In order to determine the depth and recommended size of our rain garden, we have to measure the infiltration rate," said Kass. "The infiltration rate is the speed at which water soaks into the soil, typically measured in inches per hour. "

Dr. Carvajal recommended testing the infiltration rate by conducting three tests and using the slowest infiltration rate as the basis for size. The proportions for recommended rain garden area and depth are provided below.

On their first infiltration test, Kass used a measuring tool and found the water drained 4½ inches in 12½ hours. For the second trial, Destiny had a very detailed measuring process and measured an infiltration rate of 18.73 centimeters in 14 hours. On their third trial, Huan measured an infiltration rate of ⅜ inches of drainage in 18 minutes.

Infiltration Rate	Recommended Rain Garden Size (total impervious area : rain garden surface area)
<½ inch per hour	6 inches deep with 20:7 size
½–1 inch per hour	8 inches deep with 5:1 size
>1 inch per hour	10 inches deep with 20:3 size



"Remember, Dr. Carvajal also said that we can reduce the recommended size (area) of the rain garden by 30% and still control 90% of the runoff," Destiny recalled. "A smaller but nearly as effective rain garden will help minimize the cost of rain garden construction and reduce the number of native plants that need to be planted."

Question: In square feet, what is the total surface area needed for the rain garden to meet all of the stated criteria to control 90% of the runoff? *Round each step and the final answer to the hundredths place, if needed. Note that 1 inch = 2.54 centimeters.*

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 1 ANSWER KEY – GRADE 7

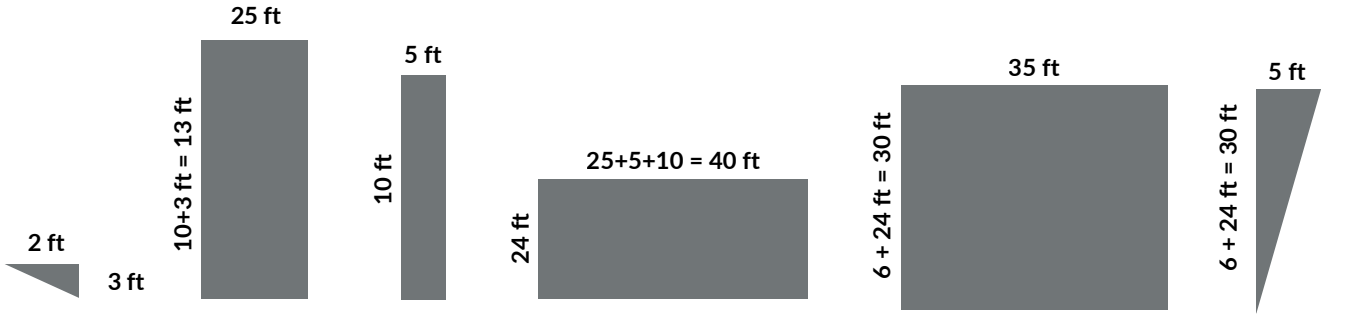
Although each problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer. Students who answered Challenge 1 correctly are invited to enter Challenge 2!

Sample Solution

Solution blueprint: My solution to find the total area needed for the rain garden to meet all of the stated criteria to control 90% of the runoff requires that I:

1. find the area of each small section of the roof.
2. find the total area of the roof.
3. figure out the slowest infiltration rate.
4. find the rain garden depth and recommended size using the infiltration rate determination.
5. find the size to account for 90% of the run-off.

Step 1: Find the area of each small section of the roof. There are a few right triangles (I can assume that they are right triangles because the description said that all angles that look like right angles are right) and the formula for area of a right triangle is ½ x (base) x (height). The formula for the area of a rectangle is (base) x (height). I was told that the image was not drawn to scale, but I have enough information to break apart individual sections.



½ x 2 x 3 = 3 square feet	13 x 25 = 325 square feet	10 x 5 = 50 square feet	24 x 40 = 960 square feet	30 x 35 = 1,050 square feet	½ x 5 x 30 = 75 square feet

Step 2: Find the total area of the roof. Adding the smaller areas found in the previous step, I can find the total surface area of the roof (which is the impervious area that will drain into the rain garden).

3+325+50+960+1,050+75 = 2,463 square feet

The total roof surface area that will be draining into the rain garden is 2,463 square feet.

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 1 ANSWER KEY – GRADE 7

(...sample solution continued...)

Step 3: Figure out the slowest infiltration rate. I need to figure out the slowest infiltration rate and so I need to know the rates in inches per hour to be able to determine the slowest rate (the least amount of water per hour drained).

Test #1 (Kass) - Kass's measurements were already in the units needed, so I just need to divide and get the unit rate to compare to the table given.

$\frac{4.5 \text{ in}}{12.5 \text{ hr}}$ = 0.36 inches/hour **Test #1 infiltration rate is <½ inch per hour.**

Test #2 (Destiny) - Destiny's measurements were in centimeters, so I will have to convert from centimeters to inches using the given conversion of 2.54 centimeters = 1 inch.

$\frac{18.73 \text{ cm}}{14 \text{ hours}} \times \frac{1 \text{ inch}}{2.54 \text{ cm}}$ = 0.5267 which rounds to 0.53 inches/hour **Test #2 infiltration rate is ½–1 inch per hour.**

Test #3 (Huan) - Huan's measurements were given in inches and minutes, so I must convert the number of minutes to hours to accurately compare knowing that 60 minutes = 1 hour.

$\frac{⅜ \text{ inch}}{18 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$ = 1.25 inches/hour **Test #3 infiltration rate is >1 inch per hour.**

The slowest infiltration rate was Test #2, which was <½ inch per hour.

Step 4: Using the infiltration rate determination, find the rain garden depth and recommended size. From the chart, with an infiltration rate of <½ inch per hour, the depth should be 6 inches deep with a ratio of total impervious area-to-rain garden size of 20:7. I know from calculations above that the size of the impervious area is 2,463 square feet.

$2{,}463 \text{ square feet of impervious area} \times \frac{7 \text{ (recommended garden size)}}{20 \text{ (impervious area)}} = 862.05 \text{ square feet}$

Based on the infiltration rate, the rain garden should be 862.05 square feet.

Step 5: Find the rain garden surface area to account for the 90% run-off. We know that we can reduce the recommended surface area of the rain garden by 30% and still control 90% of the runoff, which means that we only need 70% of the surface area of the rain garden found above.

0.70 x 862.05 = 603.435 square feet, rounded to the nearest hundredth is 603.44 square feet

The rain garden only needs to have a surface area of 603.44 square feet to control 90% of the runoff.

Final Answer: The rain garden needs to have a surface area of **603.44 square feet** to control 90% of the runoff and meet the requirement of accommodating the slowest infiltration rate.