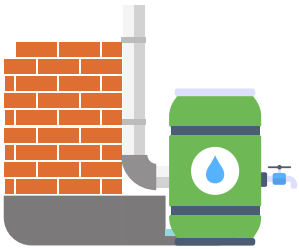
Try Your

THE HARDEST MATH PROBLEM

GRADE 8

As part of the community's initiative to empower residents and strengthen resilience against extreme weather events, guest speaker Dr. Ailem Carvajal addressed students at the South Haven Middle School (SHMS) assembly, sharing best practices in rainwater management, erosion prevention, and water bill reduction strategies that alleviate financial stressors on households.

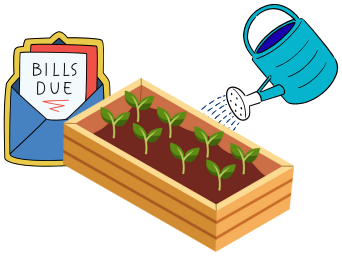


"Our state and municipality encourage residents to practice water conservation by using rain barrels to collect rainwater," said Dr. Carvajal. "Rainwater is not considered potable, which means it is not safe for human consumption, because it has not been treated and could contain residues such as bird droppings. However, rainwater is suitable for non-potable uses such as gardening, landscaping, or washing cars. Collecting rainwater can reduce a household's reliance on the municipal water system, reduce water bills, prevent storm water runoff issues like soil erosion, flooding, or water pollution, and can be healthier for watering outdoor plants."

Solve the Problem

Riley was inspired to explore water conservation options for his home and entire neighborhood. The subdivision where Riley's home is located has 25 homes and relies on the city water supply. Currently, none of the residents in Riley's subdivision utilize any kind of additional water conservation systems. In preparation to propose his water conservation idea to his neighbors, Riley met with South Haven Utilities Director, Ms. Aiyana Thunderhawk, to learn more about water usage in South Haven.

"During the summer months, the average household uses about 375 gallons of water daily where approximately 30% is used outdoors," Ms. Thunderhawk shared. "A rain barrel is a great way to supplement a household's outdoor non-potable water usage in the summer months. Our community experiences an average of 12 days of ½-inch rainfall in each summer month, regardless of whether that month has 30 or 31 days. A resident's water bill is computed by multiplying the water rate, w, (where w = the dollar amount per 1,000 gallons) by the number of gallons used."



Ms. Thunderhawk shared that the city has a new program to encourage residents to practice water conservation by offering a 3% discount on the monthly water bill (for each summer month) for homes that use a water conservation system. Residents can apply for the discount once they show that they have attached a rain barrel (or other device) to their gutter. While the water bill incentive is appealing, Riley recognized that the initial costs to purchase and install a rain barrel can be a deterrent to getting his neighbors on board with the idea.

"If I can create a flier showing savings over five years (assuming that water costs remain the same), it may help convince my neighbors to look past the initial expense and invest in a rain barrel," thought Riley. He acquired his family's most recent water bills (Bill #1: $59.58 for 9,000 gallons and Bill #2: $69.51 for 10,500 gallons) that are representative of his neighborhood's rate.

According to Dr. Carvajal, ½-inch of rain on a 200 square foot roof yields about 60 gallons of stormwater runoff. It can be assumed that the rain barrel would be completely filled and emptied after each rainfall for outdoor non-potable water use. Riley calculated that each of the neighborhood households would have a 300 square foot section of their roof that would drain into the rain barrel. Riley found that the extra-large rain barrel kits that could accommodate the ½-inch rainfall on his neighborhood's roofs were $35 each at the local hardware store with a discount of 15% if it is the first stormwater management device for a home (there is no sales tax assessed on rain barrels).

Question: After accounting for the cost of installing one rain barrel in Year 1 before summer, what is the anticipated total water bill savings for a household in Riley's neighborhood for the summer months (June, July, and August) over the first 5 years of use?
Round the final answer to the hundredths place; do NOT round any intermediate steps (i.e., use all decimal places available until the final step).

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 1 ANSWER KEY – GRADE 8

Although each problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer. Students who answered Challenge 1 correctly are invited to enter Challenge 2!

Sample Solution

Solution blueprint: My solution to find the total anticipated savings over 5 years requires me to:

1. find the rate of water collection for a neighborhood roof per rainfall day.
2. find the number of gallons expected to be collected from rainwater over the 3 summer months.
3. calculate the total number of gallons of water used over the summer months.
4. compute the water rate, *w*.
5. calculate the summer water costs without the rain barrel usage.
6. calculate the summer water costs with the rain barrel usage (which includes the city water bill discount).
7. calculate the cost of the rain barrel.
8. calculate the total summer savings over 5 years.

Step 1: Find the rate of water collection for the neighborhood roofs per rainfall. I know that a 200 square foot roof collects 60 gallons of stormwater runoff during each ½-inch rainfall. For the roofs in Riley's community, a 300 square foot section of the roof will be directed to the rain barrel, so the number of gallons of rainwater collected will be found by multiplying the roof size by collection rate.

$$300 \text{ square feet} \times \frac{60 \text{ gallons}}{200 \text{ square feet}} = \frac{18{,}000}{200} \text{ gallons} = 90 \text{ gallons}$$

A 300 square foot roof would collect 90 gallons of rainwater per rainfall day.

Step 2: Find the number of gallons expected to be collected from rainwater over the summer months. I know that there are 12 days of ½-inch rainfall expected in each of the 3 summer months (regardless of whether the month has 30 or 31 days in it). I just computed that 90 gallons of rainwater are expected to be collected in each of those days in the step above and will use that to multiply by the total number of summer rainfall days.

$$3 \text{ months} \times \frac{12 \text{ days}}{1 \text{ month}} = 36 \text{ rainfall days in the 3 summer months}$$

$$36 \text{ rainfall days} \times \frac{90 \text{ gallons of water}}{1 \text{ rainfall day}} = 3{,}240 \text{ gallons of water}$$

For the 3 summer months, 3,240 gallons of rainwater are collected.

Step 3: Calculate the total number of gallons of water used over the summer months. The rate of 375 gallons of water used daily was given, so for the three summer months I need to multiply the number of days per month by 375 and add the monthly values to get the total gallons of water used for the summer.

June: 375 gallons of water daily x 30 days = 11,250 gallons of water
July: 375 gallons of water daily x 31 days = 11,625 gallons of water
August: 375 gallons of water daily x 31 days = 11,625 gallons of water

Summer usage: 11,250 + 11,625 + 11,625 = 34,500 gallons of water

A total of 34,500 gallons of water is used by a household over the 3 summer months.

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 1 ANSWER KEY – GRADE 8

(...sample solution continued...)

Step 4: Compute the water rate, *w*. Since I was told that the water cost rate is given as dollars per 1,000 gallons, I can find the rate multiple ways from the water bills for Riley's family.

$\frac{\$69.51}{10{,}500} = \$0.00662$ per gallon or $\frac{\$59.58}{9{,}000} = \$0.00662$ per gallon $= \frac{\$6.62}{1{,}000 \text{ gallons}}$

The cost of water is $6.62 per 1,000 gallons used.

Step 5: Calculate the total summer water costs without the rain barrel usage. Without a rain barrel being used, all 34,500 gallons of water used by a household would have to come from the city water. The price for city water was computed to be $6.62 for 1,000 gallons in the previous step.

$$34{,}500 \text{ gallons} \times \frac{\$6.62}{1{,}000 \text{ gallons}} = \$228.39$$

A household without a rain barrel can expect to pay $228.39 for their 3 summer months water bill.

Step 6: Calculate the total summer water costs with the rain barrel usage (which includes the city water bill discount). When using the rain barrel, while it is anticipated that 34,500 gallons of water will be needed over the summer, 3,240 of those gallons will be the rainwater collected. Thus, I can subtract the gallons from the rain barrel from the total that will needed to find the number of gallons that will actually be needed from the city water source. Then, I can multiply the actual number of gallons needed by the water rate, *w*.

34,500 gallons needed – 3,240 gallons from rainwater = 31,260 gallons of water needed from city water

$$31{,}260 \text{ gallons} \times \frac{\$6.62}{1{,}000 \text{ gallons}} = \$206.9412$$

The bill would be $206.9412 before the discount (not rounding yet).

The city offers a 3% discount on the water bill for the summer months. I could compute this monthly and add them together, or since the question is looking at total summer savings, I can use just the total summer bill computed above for computing the discount. A 3% discount can be written as a decimal (0.03) for the calculation. After computing 3% of $206.9412 (the discount), I will subtract the discount from the $206.9412 for the actual bill amount. *Alternatively, I could just compute what 97% of the bill would be to avoid a second step of subtraction.*

$206.9412 x 0.03 = $6.208236

Subtracting the discount from the summer bill:
$206.9412 – $6.208236 = $200.732964

A household's water bill for the 3 summer months with the rain barrel (with the 3% discount) is $200.732964 (no rounding yet).

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 1 ANSWER KEY – GRADE 8

(...sample solution continued...)

Step 7: Calculate the cost of the rain barrel. The rain barrel costs $35 but has a 15% discount. *Alternatively, I could just compute what 85% of the cost would be to avoid a second step of subtraction.*

$35 x 0.15 = $5.25 off

So the rain barrel would cost: $35-$5.25=$29.75

With the 15% discount, the rain barrel would cost $29.75.

Step 8: Calculate the total summer savings over 5 years. Total savings can be computed by identifying the yearly savings by subtracting:

[(what they would have paid without the rain barrel) – (what they pay with the rain barrel)]

So each year, $228.39 – $200.732964 = $27.657036 savings annually

$27.657036 x 5 years = $138.28518 saved on the water bill for 5 years

However, purchasing and setting up the rain barrel had an initial cost (which is not savings) so to find the actual savings, the cost of the rain barrel should be subtracted from the 5-year savings total.

$138.28518 – $29.75 = $108.53518, rounding to the hundredths place: $108.54

$108.54 total water bill savings over 5 years

Final Answer: After accounting for the cost of the rain barrel, a household would expect to save a total of **$108.54** on their water bill over 5 years.