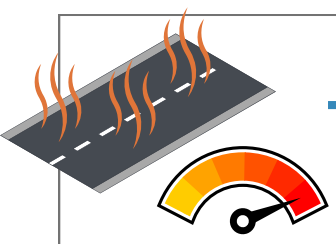


Try Your

THE HARDEST MATH PROBLEM

GRADE 8

The city of South Haven has experienced an increase in extreme heat over the past few years that has put additional strain on the city's aging infrastructure and the citizens' health. Extreme heat can pose health concerns for people (especially for very young, elderly, or vulnerable people) as well as animals. In urban areas, paved surfaces without adequate tree canopy (layers of branches, leaves, and stems that form a protective, shaded area from above) can be particularly affected by extreme heat. Dark surfaces such as pavement can absorb and radiate heat, increasing the temperature. This can lead to the phenomenon known as the "urban heat island effect".

One way to improve community resilience to extreme heat is by installing cool pavement around the city. Cool pavement is made from lighter-colored materials and is more reflective. Mr. Yilmaz, the city planner, shared with students in the South Haven Youth Civics program that the city planner's office uses a 'cooling points' system when evaluating the impact of cool pavement. "Each square meter of cool pavement earns cooling points based on three things: how much it lowers temperatures, how many people are affected, and whether those people are especially at risk from heat (like young kids, older adults, or families without air conditioning)," Mr. Yilmaz said. "Projects with total cooling point values of 200 or more are considered a significant improvement to extreme heat resilience."

Solve the Problem

Kass and Bianh have been working with the South Haven Youth Civics program after school. South Haven is launching a Cool Pavement Project to fight extreme heat, and plans to install new cool pavement in the Alvada and Burket neighborhoods. Mr. Yilmaz has asked Kass and Bianh to determine how many square meters of cooling pavement should be installed in each neighborhood to maximize the overall cooling benefit within the project's constraints.

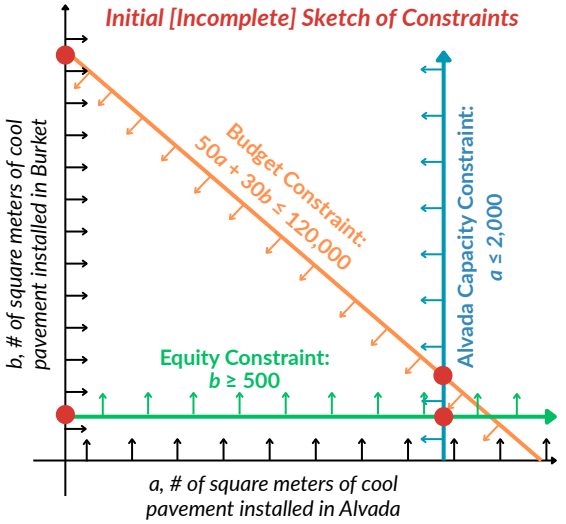
"Alvada has heavy pedestrian traffic and dense buildings with lots of paved surfaces that can intensify the urban heat island effect and has been assigned a value by the city council of 0.09 cooling points per square meter of cool pavement installed," Bianh shared. "For Burket, the wide roads and spread out housing with greater tree canopy coverage results in a score of 0.07 cooling points per square meter of cool pavement installed."

Mr Yilmaz noted that the city council has approved a total $120,000 budget for the project. Streets in Alvada are busier and require stronger materials, complicated traffic control, and night work, so every square meter of cooling pavement installed in Alvada costs $50. The Burket neighborhood is more residential with simpler streets and infrastructure, so installation is cheaper at just $30 per square meter. Additionally, the city's Cool Pavement Project only has one paving crew dedicated to resilience projects this year. Given the number of workers and the length of the warm, dry construction window, they estimate they can install at most 3,500 total square meters of cool pavement.

Kass talked with the traffic engineer about the capacity for work that could be completed in each neighborhood. "With lots of utility work and a big summer festival, up to 2,000 square meters of cooling pavement could be installed this year in Alvada," Kass said. "The target streets have been mapped out with a maximum of 3,000 square meters that can be installed in Burket this year."

Burket has historically been underinvested in and many residents don't have air conditioning. The South Haven equity officer says "We're not approving any plan that gives everything to Alvada. Burket has vulnerable residents and we require at least 500 square meters of cool pavement be installed in Burket."

Kass and Bianh began sketching a graph of a few of the constraints of the project with arrows on the constraint boundaries indicating the relevant inequalities. "For linear objective function like the cooling points, we know that the best possible solution must occur at the corner points of the shape that is formed by the boundaries of all the given constraints," Bianh shared. "And, of course, we can't install negative square meters of cooling pavement."



Question: After maximizing the total number of cooling points possible within the constraints of the project for square meters of cool pavement installed in both neighborhoods, how much of the Cool Pavement Project budget (in dollars) would be spent in the Burket neighborhood and what would be the total cool point value be for the entire Cool Pavement Project? *Round any steps and the final answer to the hundredths place, if needed.*

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY – GRADE 8

Although each problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Sample Solution

Solution blueprint: My solution to find how much of the city's budget would be spent in the Burket neighborhood and what would be the resulting cool point score for the entire Cool Pavement Project would be requires me to:

1. Write equations or inequalities for the remaining constraints
2. Graph the remaining constraints
3. Identify the points of intersection of the constraints
4. Write the function for finding the cool point value for a potential solution
5. Find the cool point value at each of the intersection points
6. Identify the maximum cool points possible
7. Find the amount of the city's budget that would be spent specifically in Burket for this point

Step 1: Write equations or inequalities for the remaining constraints. The graph that Bianh was sketching used a = # of square meters of cooling pavement installed in Alvada and b = # of square meters of cooling pavement installed in Burket. The sketch already identified:

- Budget Constraint: $50a + 30b \leq 120{,}000$ which includes the costs of each square meter of cooling pavement in each neighborhood constrained by the total budget
- Alvada Capacity Constraint: $a \leq 2{,}000$ since no more than 2,000 square meters of cooling pavement can be installed in Alvada
- Equity Constraint: $b \geq 500$ as the equity officer indicates that they won't approve a plan that doesn't address historical inequity in development in Burket so they require at least 500 square meters of cooling pavement installed in Burket
- Non-negative constraints: $a \geq 0$ since the number of square meters of cooling pavement installed cannot be negative (though it could be 0). I already know $b \geq 500$ from the equity constraint.

Still need to be sketched:

- Labor Constraint: I was told that due to the capacity of labor available for installation, the total number of square meters installed in both locations combined is limited to at most 3,500 square meters since there is just 1 crew
  - **Labor Constraint: $a + b \leq 3{,}500$**
- Burket Capacity Constraint: I was also told that given the streets that were mapped out in Burket, at most 3,000 square meters of cooling pavement could be installed in Burket
  - **Burket Capacity Constraint: $b \leq 3{,}000$**

Step 2: Graph the remaining constraints. Graphing all of the constraints together on one graph reveals that there is a space where all of the constraints are "pointing towards". As stated in the problem description, "the optimal solution must occur at the extreme intersections of the boundary lines of the given constraints".

Try Your

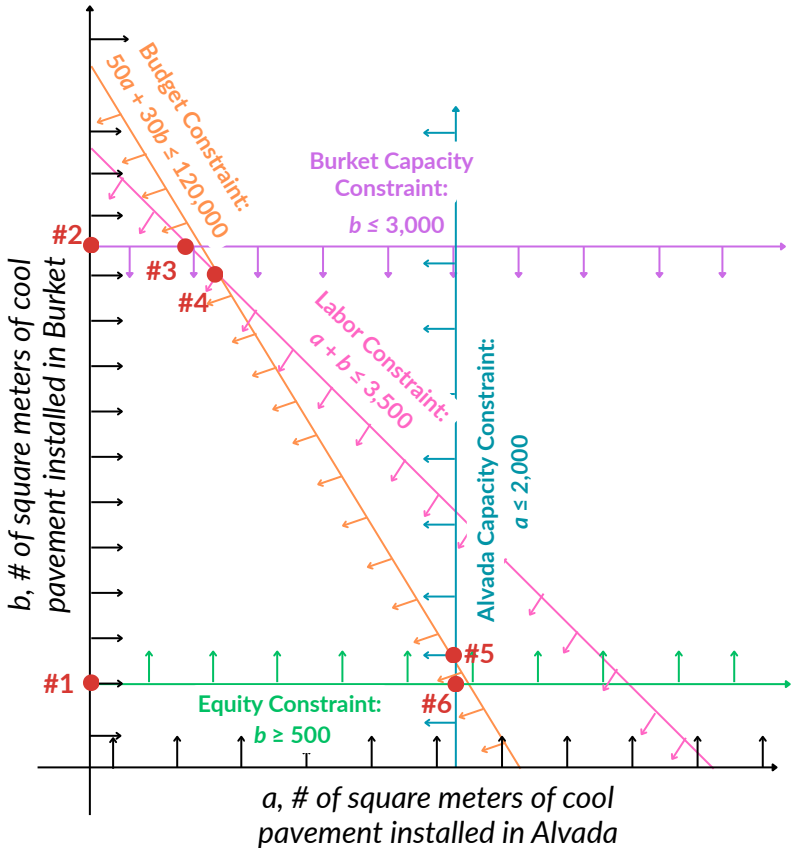
THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY – GRADE 8

(...sample solution continued...)

Step 2, continued ...
There are 6 points at which the constraints intersect and form a boundary on all points that would give a valid solution:

- Point #1: Equity constraint & the non-negative constraint
- Point #2: Burket capacity constraint & the non-negative constraint
- Point #3: Burket capacity constraint & labor constraint
- Point #4: Budget constraint & labor constraint
- Point #5: Alvada capacity constraint & budget constraint
- Point #6: Alvada capacity constraint & equity constraint



Step 3: Identify the points of intersection of the constraints. There are 6 points at which the constraints intersect. Each point of intersection occurs where the inequalities are equal, so for the purpose of solving, I'll change each inequality to an "=". Every point is in the format (a, b).

- Point #1: Equity constraint (b ≥ 500) & the Alvada non-negative constraint ($a \geq 0$)
  - b=500 and a=0 → **Point #1: (0, 500)**
- Point #2: Burket capacity constraint ($b \leq 3{,}000$) & the Alvada non-negative constraint ($a \geq 0$)
  - b=3,000 and a=0 → **Point #2: (0, 3,000)**
- Point #3: Burket capacity constraint ($b \leq 3{,}000$) & labor constraint ($a + b \leq 3{,}500$)
  - b=3,000 and $a+b$=3,500. This requires me to substitute b=3,000 into the labor constraint:
    a + 3,000 = 3,500
    −3,000 −3,000
    a = 500
    → **Point #3: (500, 3,000)**

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY – GRADE 8

(...sample solution continued...)

Step 3, continued ....

- Point #4: Budget constraint ($50a + 30b \leq 120{,}000$) & labor constraint ($a + b \leq 3{,}500$)
  - $50a + 30b = 120{,}000$ and $a + b = 3{,}500$. This is a system of equations that could be solved using elimination or substitution, I will use substitution.
    $a + b = 3{,}500$, solve for a: $a = 3{,}500 - b$
    Substitute $3{,}500 - b$ for a in the budget equation:
    $50(3{,}500 - b) + 30b = 120{,}000$
    $175{,}000 - 50b + 30b = 120{,}000$
    $175{,}000 - 20b = 120{,}000$
    $-175{,}000 \quad -175{,}000$
    $\frac{-20b}{-20} = \frac{-55{,}000}{-20}$
    $b = 2{,}750$
    Now, substitute $b=2{,}750$ back in to solve for a:
    $a=3{,}500 - 2{,}750$
    $a=750$
    → **Point #4: (750, 2,750)**
- Point #5: Alvada capacity constraint ($a \leq 2{,}000$) & budget constraint ($50a + 30b \leq 120{,}000$)
  - $a = 2{,}000$ and $50a + 30b \leq 120{,}000$, substitute $a = 2{,}000$ into the other equation
    $50(2{,}000) + 30b = 120{,}000$
    $100{,}000 + 30b = 120{,}000$
    $-100{,}000 \quad -100{,}000$
    $\frac{30b}{30} = \frac{20{,}000}{30}$
    $b = 666.67$ (rounded)
    → **Point #5: (2,000, 666.67)**
- Point #6: Alvada capacity constraint ($a \leq 2{,}000$) & equity constraint ($b \geq 500$)
  - $a = 2{,}000$ and $b = 500$ → **Point #6: (2,000, 500)**

Step 4: Write the function for finding the cool point value for a potential solution. I'm told that each square meter of cool pavement installed in Alvada has a score of 0.09 cooling points and in Burket 0.07 cooling points are assigned for each square meter of cool pavement installed. I already have established that a = # of square meters of cool pavement installed in Alvada and b = # of square meters of cool pavement installed in Burket. Therefore, the total number of cooling points that can be earned in both neighborhoods can be found with the following equation:

cooling points = $0.09a + 0.07b$

This is the function that I am trying to maximize for the most positive impact by installing cooling pavement.

Try Your

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY – GRADE 8

(...sample solution continued...)

Step 5: Find the cool point value at each of the intersection points. Each of the points of intersection that I found in step 2 are a potential optimal solution (the maximum) for the number of square meters of cooling pavement installed in each neighborhood that are within the constraints of the problem. If I plug the values for *a* and *b* from each point into the cooling point equation, I can identify the cooling point score for each potential solution and then identify the maximum.

Point #1: (0, 500) → cooling points = 0.09(0) + 0.07(500) → 0 + 35 = 35 cooling points for installing no cooling pavement in Alvada and 500 square meters in Burket.

Point #2: (0, 3,000) → cooling points = 0.09(0) + 0.07(3,000) → 0 + 210 = 210 cooling points for installing no cooling pavement in Alvada and 3,000 square meters in Burket.

Point #3: (500, 3,000) → cooling points = 0.09(500) + 0.07(3,000) → 45 + 210 = 255 cooling points for installing 500 square meters of cooling pavement in Alvada and 3,000 square meters in Burket.

Point #4: (750, 2,750) → cooling points = 0.09(750) + 0.07(2,750) → 67.5 + 192.5 = 260 cooling points for installing 750 square meters of cooling pavement in Alvada and 2,750 square meters in Burket.

Point #5: (2,000, 666.67) → cooling points = 0.09(2,000) + 0.07(666.67) → 180 + 46.67 (rounded) = 226.67 cooling points for installing 2,000 square meters of cooling pavement in Alvada and 666.67 square meters in Burket.

Point #6: (2,000, 500) → cooling points = 0.09(2,000) + 0.07(500) → 180 + 35 = 215 cooling points for installing 2,000 square meters of cooling pavement in Alvada and 500 square meters in Burket.

Step 6: Identify the maximum cool points possible. From all of the possible points for the optimal solution identified in the previous step, Point #4: (750, 2,750) → 260 cooling points had the maximum for installing:

750 square meters of cooling pavement in Alvada and 2,750 square meters in Burket.

As a check, I will make sure that this solution actually satisfies all of the constraints:

- Budget Constraint: \$50(750) + \$30(2,750) = \$37,500 + \$82,500 = \$120,000, which is indeed ≤\$120K.
- Alvada Capacity Constraint: The number of square meters installed in Alvada is *a* = 750, which is ≤ 2,000.
- Equity Constraint: b = 2,750 is indeed ≥ 500, so this does meet the requirement of at least 500 square meters of cooling pavement installed in Burket.
- Non-negative constraints: 750 ≥ 0 and 2,750 ≥ 0, so the amount of cooling pavement installed cannot be negative (though it could be 0).
- Labor Constraint: 750 + 2,750 = 3500, so the total number of square meters installed in both locations combined is indeed ≤ 3,500.
- Burket Capacity Constraint: The number of square meters installed in Burket is 2,750 which is ≤ 3,000.

This solution does indeed satisfy all of the constraints of the Cool Pavement Project.

Step 7: Find amount of the city's budget that would be spent specifically in Burket for this point. We know that 2,750 square meters of cooling pavement in Burket and it costs \$30 per square meter to install in Burket.

\$30 x 2,750 square meters = \$82,500 spent installing cooling pavement in Burket.

Final Answer: The maximum cooling points for the Cool Pavement Project is **260 cooling points** with **\$82,500** of the budget was spent in Burket.